AD-A243 159

NAVAL POSTGRADUATE SCHOOL
Monterey, California

DTIC
ELECTE
DEC 0 9 1991
S
B
D

THESIS

OPTIMAL ASSIGNMENT OF MARINE RECRUITS
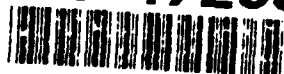TO OCCUPATIONAL TRAINING

by

Wolfgang F. Maskos

March 1991

Thesis Advisor: Richard E. Rosenthal

Approved for public release; distribution is unlimited.

91-17263

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified		1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE		Approved for public release; distribution is unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School	6b. OFFICE SYMBOL (If applicable) 55	7a. NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000		7b. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)		10. SOURCE OF FUNDING NUMBERS

Program Element No.	Project No.	Task No.	Work Unit Accession Number

11. TITLE (Include Security Classification)
OPTIMAL ASSIGNMENT OF MARINE RECRUITS TO OCCUPATIONAL TRAINING

12. PERSONAL AUTHOR(S) Maskos, Wolfgang F.

13a. TYPE OF REPORT Master's Thesis	13b. TIME COVERED From To	14. DATE OF REPORT (year, month, day) March 1991	15. PAGE COUNT 54

16. SUPPLEMENTARY NOTATION
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17. COSATI CODES			18. SUBJECT TERMS (continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUBGROUP	Optimization: Integer programming, networks, elastic constraints, multiobjective optimization. Military application: Manpower.

19. ABSTRACT (continue on reverse if necessary and identify by block number)
This thesis presents a computer based multiobjective optimization model to help Manpower Management Enlisted Assignment Branch at Headquarters Marine Corps to assign Marine recruits to occupational training.
The model is a zero-one integer program. It is solved in two phases: In phase one an elasticized linear program with relaxed integrality constraints is solved to calculate the optimal fill of the training classes. These optimal values are used to compute integer lower and upper bounds on the fill of the classes for a network model which is solved in phase two, yielding an integer solution. The model is implemented in GAMS. It was tested with real data of 461 recruits and 65 training classes on a mainframe computer and on 386/486 based Personal Computers.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS REPORT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Richard. E. Rosenthal	22b. TELEPHONE (Include Area code) (408) 646-2795	22c. OFFICE SYMBOL 55Rl

DD FORM 1473, 84 MAR   83 APR edition may be used until exhausted   SECURITY CLASSIFICATION OF THIS PAGE
All other editions are obsolete   Unclassified

i

Optimal Assignment of Marine Recruits
to Occupational Training
by

Wolfgang F. Maskos
Major, German Army
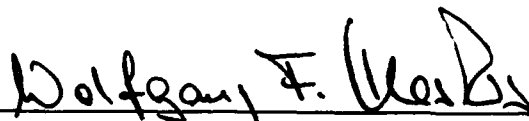Dipl.Math., University of Cologne (GE), 1978

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
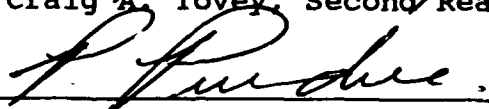March 1991

Author: _____

Wolfgang F. Maskos

Approved by: _____

Richard E. Rosenthal, Thesis Advisor

_____

Craig A. Tovey, Second Reader

_____

Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

This thesis presents a computer based multiobjective optimization model to help Manpower Management Enlisted Assignment Branch at Headquarters Marine Corps to assign Marine recruits to occupational training. The model is a zero-one integer program. It is solved in two phases: In phase one an elasticized linear program with relaxed integrality constraints is solved to calculate the optimal fill of the training classes. These optimal values are used to compute integer lower and upper bounds on the fill of the classes for a network model which is solved in phase two, yielding an integer solution. The model is implemented in GAMS. It was tested with real data of 461 recruits and 65 training classes on a mainframe computer and on 386/486 based Personal Computers.

iii

Accession For	
NTIS GRA&I	☑
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

TABLE OF CONTENTS

v

ACKNOWLEDGMENTS

Sincere thanks go to Erwin Kalvelagen of the GAMS Development Corporation for making it possible to run my model on an IBM R6000 computer.

Special thanks go to Professor Richard E. Rosenthal of the Naval Postgraduate School for his strong support of my work and his most valuable advices.

Thanks also to Major W.T. McCullers of Headquarters Marine Corps for giving me insight into the recruit assignment problem and for providing data.

I. INTRODUCTION

A. BACKGROUND

Personnel assignment problems in the armed forces usually have two major objectives: Fill as many vacant positions as possible, and fill the positions with the most qualified persons. Different positions require different qualifications, and because of more and more sophisticated weapons and equipment, a large percentage of military personnel needs occupational training to acquire these qualifications. To minimize training time and cost while furthering the second objective, individuals must be assigned to positions in a manner which makes optimal use of their existing and potential abilities.

In this connection the initial assignment of recruits to occupational training is of special importance. It is not only to be seen under the above mentioned objectives but also with respect to its influence on indivi‚al careers and job satisfaction by 'pushing the recruit in a certain direction'. Therefore, the decision process determining the initial assignment must not be based on rule-of-thumb procedures and/or intuition but on effective alternatives resulting from a profound problem analysis.

The Manpower Management Enlisted Assignment (MMEA) Branch of Headquarters United States Marine Corps (HQMC) faces this decision process about once a week, when on average 750 recruits are to be distributed among 60 or more training classes after graduation from the School of Infantry (SOI).

B. GOALS OF THE THESIS

The purpose of this thesis is to develop a prototype computer system based on a mathematical optimization model which can be used to help MMEA assign Marine recruits to occupational training. It must have the following properties:

1. The model should give realistic answers which can be directly usable for nearly all recruits considered.

2. The model should be as generic as possible. An increase in problem size (e.g. more recruits) must not result in any model changes or in extremely degraded performance.

3. Only integer solutions are acceptable.

4. The model must be easy to modify in order to accommodate changes for future concerns. For this reason, it should use general purpose solvers, rather than rely upon algorithms designed for a unique purpose.

5. The results of the model must be directly importable into other software packages (e.g. spreadsheets).

6. The implementation should not be hardware specific. The model should run on mainframes, workstations, and 386/486 Personal Computers, and it should also be implementable in future computer environments.

C. TERMINOLOGY

This section describes terminology used in the model.

1. Prerequisites

When recruits are selected for training classes, their existing qualifications are compared with desired qualifications which are defined for each class. The desired qualifications are called prerequisites. We distinguish between mandatory and desirable prerequisites: Mandatory prerequisites describe the minimum qualifications which a recruit must meet to be eligible for a training class, while desirable prerequisites define qualifications which a recruit should have in addition to the mandatory prerequisites, in order to be a more desired candidate for a training class.

2. Prerequisite Level

For each training class the prerequisites are combined in one or more sets, where each set represents a level of desired qualification of the trainees. The sets are called prerequisite levels. The levels build a hierarchy of quality and are ranked by integers, starting with one for the set of mandatory prerequisites as the lowest level. In order to satisfy prerequisite level n of a class a recruit must have not only the qualifications described in level n, but also the qualifications of all lower levels n-1, n-2, ..., 1. A recruit who is not eligible for a class is said to have

prerequisite level zero for this class. The number of specified prerequisite levels can differ between classes.

3. Quota

The number of seats in a training class is called a quota. The quota is the largest and at the same time the most desirable number of trainees to enroll in a class.

4. Fill Priority

The fill priority of a training class is an integer which ranks the relative importance of filling the class. Fill priorities are necessary inputs because of the frequently occurring possibility that not all quotas can be met. Classes with the same fill priority form a priority group.

5. Fit Priority

The fit priority of a training class is an integer which ranks the relative importance of filling the class with recruits having high prerequisite levels.

6. Area Aptitude Composite

Each Marine recruit must pass the Army Area Aptitude Battery in which his or her intellectual, psychic, and motor abilities are tested. The Area Aptitude Composites (AAC) are linear combinations of the scores a recruit received in the basic tests of the battery. The AACs are used to estimate a recruit's success in the training classes. For each class one relevant AAC is specified.

D. THESIS OUTLINE

In Chapter II, the Marine Corps objectives and policies for recruit assignments are presented. The current solution and data sources are also described in this chapter.

A basic recruit assignment model is formulated in Chapter III. The difficulties in finding feasible, integer solutions to this model are also discussed. The basic model is not presented as a viable approach, but as a valuable initial framework. Chapter IV contains the redevelopment of the basic model into a two phase model. The recruit assignment problem is decomposed into a linear programming subproblem and a network subproblem , which are solved in sequence.

The computer implementation of the model, preprocessing of input data, and results are described in Chapter V. Conclusions and recommendations for future improvement are given in Chapter VI.

5

II. ASSIGNMENT OF MARINE RECRUITS TO OCCUPATIONAL TRAINING

A. OBJECTIVES OF HEADQUARTERS USMC

The Manpower Management Enlisted Assignment Branch at Marine Headquarters has four objectives for the recruit assignment problem:

1. Maximize the fill of the training classes. If quotas cannot be met, allocate vacancies according to the fill priorities.

2. Maximize the quality of assignments as measured by prerequisite level, with competition for high quality recruits adjudicated by the fit priorities.

3. Minimize the total waiting time between the recruits' graduation from Infantry School and the beginning of training classes.

4. Maximize the expected success of the recruits by assigning each recruit as close as possible to the training class for which he has the highest proficiency as measured by the Area Aptitude Composite (AAC).

The objectives are listed above in their current order of importance. The models developed in this thesis would allow for the order to change.

Objectives 2 and 4 are often correlated, since for all training classes a minimum score in the relevant AAC is a mandatory prerequisite, and for most training classes the required score increases with the prerequisite level. On the other hand, these objectives can also be conflicting. A recruit might not be eligible for the training classes

corresponding to his or her highest AAC, because he or she does not meet other mandatory prerequisites of the classes.

The waiting time objective depends on the assignment decisions because the recruits can have different availability dates and the training classes have different start dates.

B. POLICIES OF HEADQUARTERS USMC

Besides the constraints that each recruit has to be assigned to exactly one training class and that each recruit must meet the mandatory prerequisites of his or her assigned class, three policies have to be observed in the recruit assignment problem.

1. Program-Enlisted-For (PEF)

A high percentage of Marine recruits are guaranteed by their recruiters that they will get a job out of a specific group of assignments. These agreements must be honored. The given PEF guarantee can possibly overrule the constraint on mandatory prerequisites for training classes.

2. Minority Distribution Policy

Recruits are classified as either minority group members or non-minorities. Each training class must adhere to a specified minimum and maximum percentage filled with minorities. The percentages can differ between training classes and can also change with time.

3. Allocation of Shortages

While the fill priorities guide the allocation of shortage of qualified recruits between priority groups, sharing coefficients determine the allocation of shortage between training classes within the same priority group. For each class j a sharing coefficient $s_j > 0$ specifies desired sharing targets within priority groups as follows:

Let classes k and j belong to the same priority group. If all quotas in this priority group cannot be fully satisfied, a desired outcome is to have classes k and j share the shortage so that the ratio of their shortage percentages is equal to the ratio of their sharing coefficients, i.e. (shortage$_k$ %) / (shortage$_j$ %) = s_k / s_j. Typically, the sharing coefficients are one, so that equal sharing of shortage within priority classes is encouraged.

C. CURRENT SOLUTION

MMEA currently uses the "Recruit Distribution Model (RDM)" which is a product of the Decision Systems Associates, Inc. (DSAI) [Ref. 1]. The model was first developed in 1965 and has been adapted by DSAI to changing objectives and policies since then. RDM consists of a system of optimizing algorithms, which are applied sequentially to the problem, generating at each stage additional constraints for the following stages. RDM

currently runs only on a Control Data Corporation CYBER 175, a mainframe computer that was introduced in the early 1970s.

The underlying mathematical model of RDM has not changed. As new concerns of the Marine Corps were encountered, many of them were incorporated through the definition of prerequisite levels.

D. DATA SOURCES

1. Recruit Data

Recruit data records are kept in the USMC Recruit Accession Management System (RAMS). These records contain the name, Social Security number and all characteristics of the recruit which are necessary to determine his or her eligibility for each training class.

2. Training Class Data

Quota, start date, and fill priority of each training class are provided in the Training Quota Memorandum (TQM), which is generated by Marine Corps Development and Education Command. Fit priorities, codes of relevant AACs, number of prerequisite levels, and the prerequisite set for each prerequisite level are maintained by MMEA in a file called the Prerequisite Dictionary.

3. Program-Enlisted-For Data

MMEA maintains a catalog of all PEF guarantees and the corresponding promised assignments. From this file we obtain the information to which training classes a recruit with PEF

guarantee can be assigned, and for which of these classes the constraint on mandatory prerequisites may be overruled.

III. BASIC RECRUIT ASSIGNMENT MODEL

A. FORMULATION OF THE MODEL

This section describes an initial translation of the recruit assignment problem into a mathematical model. This model is not regarded as a viable approach on its own, but it serves as a framework upon which a usable model can be built.

Indices

$i \in I$ - available recruits

$j \in J$ - training classes

Sets

C_i - set of all classes for which recruit i is eligible

R_j - set of all recruits who are eligible for class j

MI   - recruits who are members of a minority

MIR_j - intersection of MI and R_j

Given Data

q_j - quota of class j (men)

$lmin_j$ - minimum minority fraction of class j

$hmin_j$ - maximum minority fraction of class j

s_j - sharing coefficient of class j

$filpr_j$ - fill priority of class j

$fitpr_j$ - fit priority of class j

aac_{ij} - score of recruit i in AAC relevant for class j

t_{ij} - time between graduation of recruit i and start of class j

pl_{ij} - prerequisite level which recruit i meets for class j

Decision Variables

x_{ij} = 1 if recruit i is assigned to class j

        = 0 otherwise

Constraints

$$\sum_{j \in C_i} x_{ij} = 1 \qquad \text{for all } i \in I \qquad (3.1)$$

Assign each recruit to exactly one class.

$$\sum_{i \in R_j} x_{ij} \leq q_j \qquad \text{for all } j \in J \qquad (3.2)$$

Observe the upper limit of seats (quota) for each class.

$$lmin_j \le \frac{\sum\limits_{i \in MIR_j} x_{ij}}{\sum\limits_{i \in R_j} x_{ij}} \le hmin_j \qquad \text{for all } j \in J \quad (3.3)$$

Observe the minority distribution for each class.

$$\frac{\dfrac{q_j - \sum\limits_{i \in R_j} x_{ij}}{q_j}}{\dfrac{q_k - \sum\limits_{i \in R_k} x_{ik}}{q_k}} = \frac{s_j}{s_k} \qquad \text{for all } j,k \in J \quad (3.4)$$

where classes j and k belong to the same priority group and k < j.

Observe the policy for share of shortage between classes with the same fill priority.

Constraints (3.3) and (3.4) can be rewritten in linear form:

$$lmin_j \sum\limits_{i \in R_j} x_{ij} \le \sum\limits_{i \in MIR_j} x_{ij} \le hmin_j \sum\limits_{i \in R_j} x_{ij} \qquad (3.3')$$

$$s_k \frac{q_j - \sum\limits_{i \in R_j} x_{ij}}{q_j} - s_j \frac{q_k - \sum\limits_{i \in R_k} x_{ik}}{q_k} = 0 \qquad (3.4')$$

$$x_{ij} \in \{0,1\} \qquad \text{for all } (i,j) \in (I,J) \quad \text{s.t. } i \in R_j \qquad (3.5)$$

The decision variables are binary variables, and are defined only if recruit i is eligible for class j.

Objective Functions

The problem has four objective functions:

$$\text{Maximize} \quad \sum_{j \in J} filw_j \sum_{i \in R_j} x_{ij} \quad\quad (3.6)$$

Maximize fill of training classes according to the fill priorities.

$$\text{Maximize} \quad \sum_{j \in J} fitw_j \sum_{i \in R_j} pl_{ij} \, x_{ij} \quad\quad (3.7)$$

Maximize quality of assignments according to fit priorities and prerequisite levels.

$$\text{Minimize} \quad \sum_{j \in J} \sum_{i \in R_j} t_{ij} \, x_{ij} \quad\quad (3.8)$$

Minimize waiting time.

$$\text{Maximize} \quad \sum_{j \in J} \sum_{i \in R_j} \frac{aac_{ij}}{maxaac_i} \, x_{ij} \quad\quad (3.9)$$

Maximize expected success of recruits.

Parameter $maxaac_i$ is the highest score recruit i achieved in all AACs ($maxaac_i = \max \{aac_{ij} \mid j \in J\}$).

Parameter $filw_j$ is the fill weight of class j and is calculated by: $filw_j = ((minfilpr + 1) - filpr)^2$, where minfilpr is the lowest fill priority. Squaring the difference

accentuates the requirement that classes with high fill priorities are filled first.

Parameter $fitw_j$ is the fit weight of class j and is calculated by $fitw_j$ = (minfitpr + 1) − $fitpr_j$, where minfitpr is the lowest fit priority.

B. DISCUSSION OF THE BASIC MODEL

1. Objective Functions

The recruit assignment problem is a multiobjective optimization problem for which various approaches exist [Ref. 2]. One approach is to specify a weight for each objective function according to its relative importance, add the weighted functions, and solve the problem using a solver for single-objective problems. This procedure will be applied to the basic model. The objective functions of the basic model have different units and differ in the size of the parameters, which means they are not comparable. Therefore, the objective functions must be transformed to a common scale before they can be weighted and added.

2. Feasibility

The model described above is seldom feasible. A necessary but not sufficient condition for feasibility is that the total number of recruits is less than or equal to the sum of the quotas. Since the quotas are often planned before enough information about the recruit resources is known, this feasibility condition can be violated.

Another source of infeasibility is the PEF policy, which heavily reduces the number of eligible recruit-class combinations. This can result in the situation that some classes have more eligible candidates than their quotas, while other classes have a drastic shortfall. The situation is made even worse by the sharing constraint (3.4), which, if enforced, would prevent all classes of the priority group from getting a higher fill percentage than the class with the least fill. Consequently, in practice, either some eligible recruits do not get an assignment or the sharing constraint is violated. Either case is infeasible in the initial model, but this situation may not be avoidable.

Also, the minority constraint may not be satisfiable because the number of qualified minority members can fall below the requested lower limit or exceed the upper limit.

The basic model has to be modified in a manner which takes the above mentioned possibilities for infeasibility into consideration and guarantees usable results. The model should choose which constraint to violate based on policy parameters entered by the user.

3. Integrality

The model is a zero-one integer program (IP). Without the minority and sharing constraints it is a network problem, for which integer solutions are guaranteed using a linear program (LP) solver [Ref. 3]. These two constraints destroy

the network structure and cause the solution of the corresponding LP to fractionate. This means that, in order to get integer solutions, either an IP algorithm or a heuristic must be used. The solution time of IP problems can drastically increase as the number of variables increases. The typical recruit distribution problem has about 750 recruits and 60 training classes with about 20% of the recruits, on average, eligible for each class. This problem size yields a model size of 9000 binary variables, which is much too large to guarantee acceptable run times, given the model's structure. Therefore, a rounding heuristic applied to the solution of the LP relaxation seems to be favorable for this problem. We have chosen an optimization-based heuristic.

In the next chapter we develop a new, practical approach which overcomes the shortcomings of the initial, basic model.

IV. DEVELOPMENT OF THE MODEL

A. ELASTIC VARIABLES AND PENALTY COST

1. Mathematical Background

A common procedure to overcome infeasibility is to elasticize (sometimes called "soften") the constraints which can cause infeasibility [Ref. 4, 5]. Elasticizing a constraint means introducing additional nonnegative variables which represent the under- or overachievement of the originally desired range of the constraint. These "elastic" variables are multiplied by penalty costs and added to the objective function.

Elasticizing constraints yields useful information when the original model is infeasible. Nonzero elastic variables in the optimal solution indicate which constraints cause infeasibility in the original model and, by extension, which parameters would have to be changed in order to move toward feasibility. When it is possible to avoid infeasibility, the elastic penalties enable the model to select which constraints to violate and by how much. It also allows the model to reflect the common practice of trading off satisfaction of one constraint for another constraint or for objective function improvement.

2. Elasticizing the Minority Constraints

The minority constraints (3.3') are elasticized by subtracting the positive variable $dminu_j$, which is the shortage of minority members in class j, from the left hand side of the inequality, and adding the positive variable $dmino_j$, which gives the number of minority members exceeding the upper minority limit of class j, to the right hand side. To keep the sum of the penalty costs of this constraint between zero and one, the penalty costs must be divided by the largest value that $dminu_j$ or $dmino_j$ can take on. That value is $maxm_j = \max \{minq_j\ q_j, (1 - hminq_j)q_j\}$. So $pmin_j$ is specified for all $j \in J$ by $pmin_j = 1 / (maxm_j\ A)$, where A is the total number of planned classes.

The new constraints are:

$$lmin_j \sum_{i \in R_j} x_{ij} - \sum_{i \in MIR_j} x_{ij} - dminu_j \leq 0 \qquad \text{for all } j \in J$$

(4.4)

Observe the lower bound on minorities for each class.

$$\sum_{i \in MIR_j} x_{ij} - hmin_j \sum_{i \in R_j} x_{ij} - dmino_j \leq 0 \qquad \text{for all } j \in J$$

(4.5)

Observe the upper bound on minorities for each class.

3. Elasticizing the Sharing Constraints

The linear sharing constraints (3.4') are elasticized by adding to the left hand side the positive variables dsu_{jk}

and dso_{jk}, which represent underachievement and overachievement, respectively, of the desired fill percentage of class k in comparison with class j. Since both violations are equally undesirable the same normalized penalty costs ps_{jk} are specified for both variables by $ps_{jk} = 1 / (smax * N)$, where N is the total number of combinations $(j,k) \in (J \times J)$, so that classes j and k belong to the same priority group and $k < j$, and $smax = max \{s_j \mid j \in J\}$. The new constraint is:

$$s_k \frac{q_j - \sum_{i \in R_j} x_{ij}}{q_j} - s_j \frac{q_k - \sum_{i \in R_k} x_{ik}}{q_k} + dsu_{jk} - dso_{jk} = 0 \qquad (4.6)$$

for all j, k $\in$ J, where class j and class k are in the same priority group and $k < j$.

Observe the policy for share of shortage between classes with the same fill priority.

4. Elasticizing the Supply Constraints

The supply constraints (3.1) are elasticized by adding to the left hand side of the equation the positive variable da_i, which allows the possibility that recruit i is not assigned to any class. When determining the penalty cost pa_i we must take into consideration that only eligible recruits can be assigned to a class, and that recruits with a PEF guarantee must get an assignment first. The following costs satisfy these conditions: $pa_i = 0$, if C_i is empty; $pa_i = 3$, if C_i is not empty and recruit i has no PEF guarantee;

$pa_i = 3 * \text{NOPEF}$, if C_i is not empty and recruit i has a PEF guarantee, where NOPEF is the number of all recruits without PEF guarantee. The penalty cost for not assigning eligible recruits is higher than the sum of all other penalty costs. This causes the model to assign eligible recruits, even if the original minority and/or sharing constraints have to be violated.

The new constraint is:

$$\sum_{j \in C_i} x_{ij} + da_i = 1 \qquad \text{for all } i \in I \qquad (4.2)$$

Assign each recruit to at most one class.

B. NORMALIZED AND WEIGHTED OBJECTIVE FUNCTIONS

1. General Idea

A widely used procedure to make conflicting objectives comparable is to score each of them on the scale [0, 1]. Let A_n be the achievement level of objective n. Define $a_n = (A_n - A_{nw}) / (A_{nb} - A_{nw})$, where A_{nb} is the most desirable and A_{nw} the least desirable achievement level of objective n, then $a_n \in [0, 1]$. This method, called "proportional scoring", will be applied to the objective functions of the basic model. The scored functions will be weighted in accordance with their relative importance.

2. Fill of Training Classes

It is most desirable that the number of assignments be equal to the sum of the quotas, and least desirable, that no recruit be assigned to any class. Therefore the fill objective is normalized by dividing by the sum of the quotas.

The fill weights $filw_j$ as described in Section III A not only determine the order in which the classes are filled, but also weight the fill objective function relative to the other objectives.

3. Fit of Assignments

The most desirable level of the fit function is achieved if all recruits in a training class have the highest prerequisite level of this class. The least desirable achievement occurs if all recruits have only the mandatory attributes, i.e., prerequisite level one. (Level zero is not possible, since those recruits are not eligible for this class.) Therefore, pl_{ij} is replaced by $(pl_{ij} -1) / ((hl_j - 1)R)$ in function (3.7), where hl_j is the highest prerequisite level of class j and R is the total number of eligible recruits. If $hl_j = 1$, class j is omitted from the fit objective, because the fit quality is not controllable for such a class.

The fit weights $fitw_j$ as described in Section III A serve two purposes. They give the classes priorities for receiving trainees with high prerequisite levels, and they

give weight to the fit objective function relative to the other objectives.

4. Waiting Time

The optimal waiting time for all assigned recruits is zero, while the worst case is $maxt = \max \{t_{ij} | \ i \in I, \ j \in J\}$. Therefore, t_{ij} is replaced by $-(maxt - t_{ij}) / (maxt \ R)$ in function (3.8).

5. Expected Success of Recruits

A recruit is most likely to succeed in the training class for which he or she has the highest proficiency, as measured by the Area Aptitude Composite. The expected success decreases as the AAC score decreases and is defined to be zero if the score is zero. For scaling, $aac_{ij} / maxaac_i$ in function (3.9) is replaced by $aac_{ij} / (maxaac_i \ R)$.

C. FORMULATION OF THE ELASTICIZED MODEL

The model with elasticized constraints can be written as follows:

Maximize

$$w_{fill} \sum_{j \in J} filw_j \sum_{i \in R_j} \frac{x_{ij}}{\sum_{j \in J} q_j} \qquad\qquad (4.1)$$

$$+ \quad w_{fit} \sum_{j \in J} fitw_j \sum_{i \in R_j} \frac{pl_{ij} - 1}{(hl_j - 1) \; R} \; x_{ij}$$

$$+ \quad w_{time} \sum_{j \in J} \sum_{i \in R_j} \frac{maxt - t_{ij}}{maxt \; R} \; x_{ij}$$

$$+ \quad w_{aac} \sum_{j \in J} \sum_{i \in R_j} \frac{aac_{ij}}{maxaac_i \; R} \; x_{ij}$$

$$- \quad w_{supp} \sum_{i \in I} pa_i \; da_i$$

$$- \quad w_{min} \sum_{j \in J} pmin_j \; (dminu_j + dmino_j)$$

$$- \quad w_{shar} \sum_{j \in J} \sum_{k \in J} ps_{jk} \; (dsu_{jk} + dso_{jk})$$

(The seven parts of the objective function are weighted as described in Section IV B. The parameters w_{fill}, w_{fit}, etc. were all set to one in our runs of the model, but they are included to give the user an additional possibility to tune the model.)

Subject to

$$\sum_{j \in C_i} x_{ij} + da_i = 1 \qquad \text{for all } i \in I \qquad (4.2)$$

Assign each recruit to at most one class.

$$\sum_{i \in R_j} x_{ij} \leq q_j \qquad \text{for all } j \in J \qquad (4.3)$$

Observe the quota of each class.

$$lmin_j \sum_{i \in R_j} x_{ij} - \sum_{i \in MIR_j} x_{ij} - dminu_j \leq 0 \qquad \text{for all } j \in J \qquad (4.4)$$

Observe the lower bound on minorities for each class.

$$\sum_{i \in MIR_j} x_{ij} - hmin_j \sum_{i \in R_j} x_{ij} - dmino_j \leq 0 \qquad \text{for all } j \in J \qquad (4.5)$$

Observe the upper bound on minorities for each class.

$$s_k \frac{q_j - \sum_{i \in R_j} x_{ij}}{q_j} - s_j \frac{q_k - \sum_{i \in R_k} x_{ik}}{q_k} + dsu_{jk} - dso_{jk} = 0 \qquad (4.6)$$

for all $j, k \in J$, where class j and class k are in the same priority group and $k < j$.

Observe the policy for share of shortage between classes with the same fill priority.

$$da_i \geq 0 \qquad \text{for all } i \in I$$

25

$$dminu_j, \ dmino_j, \ dsu_{jk}, \ dso_{jk} \ \geq 0 \qquad \text{for all } j, k \in J$$

$$x_{ij} \in \{0, 1\} \qquad \text{for all } i \in I, \ j \in J, \ s.t. \ i \in R_j.$$

D. TWO-PHASE APPROACH

1. Motivation

The elasticized model is still an IP problem for which an LP solver is very likely to yield fractional solutions. To get completely integer solutions, the following two-phase approach is taken:

In the first phase, the LP relaxation of the elasticized model is solved. This means the integrality constraint is replaced by x_{ij} is greater than or equal to zero. The optimal solution of the LP relaxation is used to calculate integer upper and lower bounds on the fill of the classes. These bounds are kept as close as possible to the relaxed problem's optimal fill values, so that, as long as the fills are varied only within the bounds, the corresponding minority and sharing ratios will also stay very close to the values found by the optimal solution of the LP relaxation.

Therefore, in phase two we can replace the sharing constraint by using the new bounds in the fill and minority constraint. This results in a network model, which guarantees completely integer LP solutions.

A similar two-phase approach was used quiet successfully in scheduling flowlines [Ref. 6].

2. Computation of the Bounds

Let

$$fill_j = \sum_{i \in R_j} x_{ij}^{(1)} \qquad \text{for all } j \in J,$$

where $x_{ij}^{(1)}$ are the optimal solutions of the LP version of the elasticized model. The lower bound of the fill of class j is then defined by loq_j = floor ($fill_j$), where floor (a) is the greatest integer smaller than or equal to a. The upper bound is defined by hiq_j = min (loq_j + 1, q_j), which makes sure that the final fill value is no more than one unit away from phase one fill, and also no greater than the original quota.

Based on these bounds the smallest and largest number of minority recruits is calculated. The lower limit is given by lom_j = floor ($lmin_j$ hiq_j), and the upper limit by him_j = ceiling ($hmin_j$ hiq_j), where ceiling (a) is defined as the smallest integer greater than or equal to a.

3. The Network Model

Mathematically the network model can be stated as follows:

Maximize

$$\sum_{j \in J} \sum_{i \in R_j} u_{ij} \, x_{ij} \quad - \quad w_{supp} \sum_{i \in I} pa_i \, da_i \qquad (4.7)$$

$$- \quad w_{min} \sum_{j \in J} pmin_j \, (dminu_j + dmino_j)$$

27

where u_{ij} is the sum of the coefficients of the first four parts of the objective function (4.1).

Subject to

$$\sum_{j \in C_i} x_{ij} + da_i = 1 \qquad \text{for all } i \in I \qquad (4.8)$$

Assign each recruit to at most one class.

$$\sum_{i \in R_j} x_{ij} \geq loq_j \qquad \text{for all } j \in J \qquad (4.9)$$

Observe the lower bound on quota for each class.

$$\sum_{i \in R_j} x_{ij} \leq hiq_j \qquad \text{for all } j \in J \qquad (4.10)$$

Observe the upper bound on quota for each class.

$$\sum_{i \in MIR_j} x_{ij} + dminu_j \geq lom_j \qquad \text{for all } j \in J \quad (4.11)$$

Observe the lower bound on minorities for each class.

$$\sum_{i \in MIR_j} x_{ij} - dmino_j \leq him_j \qquad \text{for all } j \in J \quad (4.12)$$

Observe the upper bound on minorities for each class.
All variables are nonnegative.

Figure 1 illustrates a network model which is equivalent to the phase two linear program. The nodes P_i represent the available recruits, M_j the minority quotas, and

Q_j the quotas of the training classes. The final node S is used in the network solution process to draw recruits through the system. Without loss of generality we can assume that nodes P_1 through P_m represent minority members while $P_{(m+1)}$ through P_r denote non-minorities. Each recruit node has a supply of one, indicating that each recruit can be assigned to only one class. Node S has demand R which is the number of all available recruits. All other nodes have supply equal zero and serve as transshipment nodes.

An arc representing x_{ij} exists if recruit i is eligible for training class j. This arc has tail P_i, and if recruit i is a minority member, it has head M_j. If the recruit is a nonminority, it has head Q_j. This arc has objective function coefficient u_{ij}, the gain of assigning recruit i to class j.

The arc from M_j to Q_j represents the required minority enrollment in class j. The lower and upper bounds on this arc are obtained from the solution of the phase one subproblem as described above. Similarly, the arc from Q_j to S represents the required total enrollment in class j. The arcs (M_j,Q_j) and (Q_j,S) have zero cost because traversing them means that constraints are being satisfied.

The elastic variable $dmino_j$, corresponding to excess minorities in class j, is represented by the arcs from minority recruits to Q_j. These arcs have an objective coefficient of $-pmin_j$ representing the elastic penalty cost
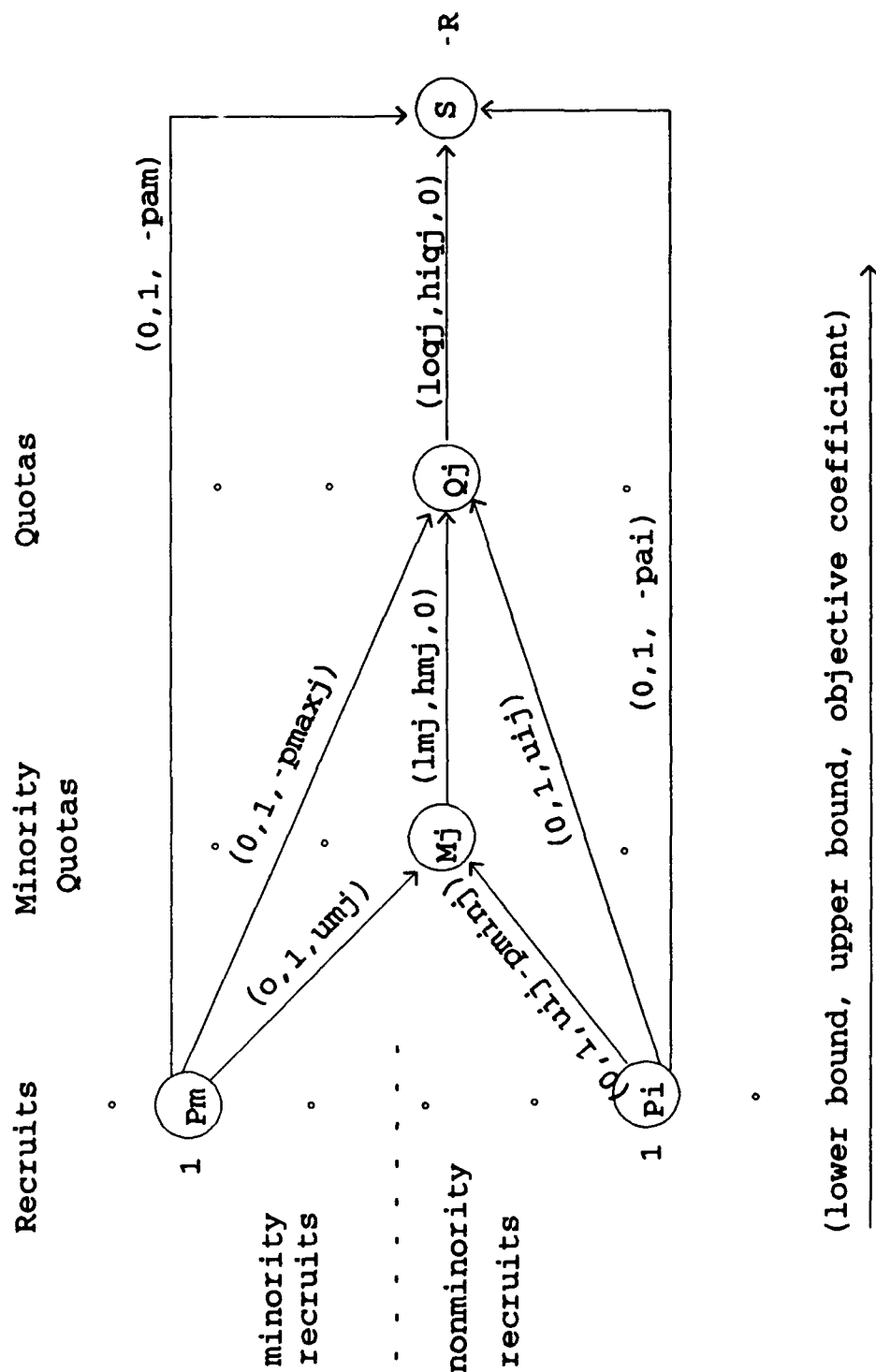
Figure 1. Network Model for Phase Two

for excess minorities. The possibility of a shortage of minorities is modeled with arcs from the nonminority recruit nodes to M_j. If this elastic arc is used, a nonminority recruit will be counted in a minority quota, for which a penalty of $pmin_j$ is charged.

Each recruit is also connected by an arc to node S, so that if recruit i cannot be assigned to any class, he will be assigned directly to node S at penalty cost pa_i.

4. Summary of the Two-Phase Model

The two-phase model is solved as follows:

1. Solve the LP relaxation of the elasticized model defined by (4.1) through (4.6).

2. Compute the lower and upper bounds on minority fill and total fill by the formulas in Subsection IV C 2.

3. Solve the network model defined by (4.7) through (4.12).

This combination of steps is guaranteed to achieve an integer solution.

'

V. COMPUTER IMPLEMENTATION AND RESULTS

A. GAMS MODELING LANGUAGE

A GAMS [Ref. 7] computer program was written to obtain optimal solutions for the two phase recruit assignment problem. The GAMS modeling language was used for several reasons:

1. Since the GAMS model representation is the same as the mathematical representation, the computer program is easy to read and understand, which is especially important if the users change frequently as is common in the military.

2. An optimization model implemented in GAMS is very easy to modify. This is important in the case the Marine Corps decides to add new constraints in the future. For example, gender distribution limits can be handled the same way as the minority constraints. For another example, a desire to spread quality recruits among classes can be handled the same way as the sharing constraints.

3. GAMS is an extremely convenient way to execute a sequence of optimization models in which the inputs to the second model depend on the optimal solution of the first model, as required by our two-phase method.

4. GAMS allows the model to be formulated and maintained independently from the data it uses. This means that data can be changed or the problem can increase in size without resulting in problem changes.

5. Due to the system design of GAMS, future solution algorithms can be used without the requirement to change the representation of the model, e.g., a faster linear program or network solver can be substituted for the current solver by changing one line of GAMS input.

6. The GAMS system does not require special input or output procedures. Input files can be written with any text editor or can be the result of other software packages.

In the recent version of GAMS, version 2.25, the GAMS program can be linked to spreadsheets. This can be used in our problem to write the recruits' assignment orders using the optimal assignments found by GAMS.

7. GAMS programs are portable from mainframes to workstations and PCs and vise versa without any changes in the program. GAMS is not designed for a special hardware.

The above described features of GAMS satisfy the goals of this thesis as specified in Chapter I.

B. PREPROCESSING OF INPUT DATA

The raw data given by the data sources described in Chapter II are preprocessed as follows:

1. From the recruit data the scores of the Area Aptitude Composites, minority membership, graduation date, and a PEF flag are written to the file RECDATA.

2. The qualifications of the recruits as given in the recruit data are compared with the prerequisites of the available training classes, which are contained in the Prerequisite Dictionary. By that we find the prerequisite level of each recruit for each class. For recruits with PEF guarantees the prerequisite levels of inappropriate classes are set to zero. The prerequisite level matrix (rows = recruits, columns = classes) is written to file PRELEV.

3. Quota, start date, and fill priority of each training class, as given in the Training Quota Memorandum, are combined with the code of the relevant AAC, fit priority, number of specified prerequisite levels, and the lower and upper limit of the minority fraction in file QUOTAS.

The first two input files are generated by a FORTRAN77 program which was written by the author, whereas the QUOTAS file was written with a text editor. A generic program which

generates the three input files would have exceeded the scope of this thesis. Figure 2 shows which raw data (rectangles) are used to generate the input data (ellipses) for the program.
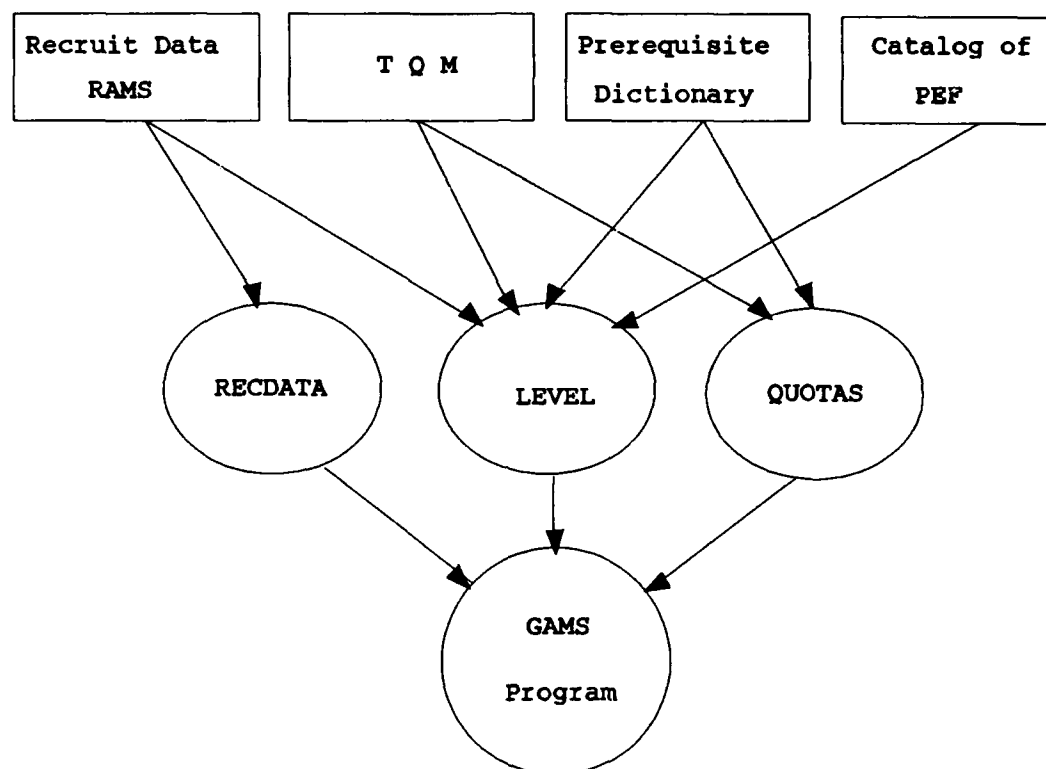

Figure 2.   Preprocessing of Input Data

Ideally, the raw data should reside in database management system (DBMS) and the GAMS program should extract appropriate data and execute via DBMS commands. Such an environment is described in "TC-EXPRESS Version of the Recruiter Allocation Model" [Ref. 8].

C. COMPUTATIONAL RESULTS

The model was tested with real data of 461 recruits and 65 training classes obtained from MMEA. The training classes had 23 different fill priorities and 9 fit priorities. The resulting GAMS model had 886 equations and 5858 variables in phase one (elasticized model), and 848 equations and 5587 variables in phase two. The relatively small number of variables, compared to 461 * 65 = 29965 theoretically possible decision variables, is due to the PEF policy which reduced the number of eligible recruit-class pairings to 4869 (density = 16.3%). This can be fully exploited in the GAMS model.

The GAMS program was run on three computers:

1. AMDAHL 5990-500 mainframe using the CP/CMS timesharing system with GAMS 2.19 and ZOOM 2.1.

2. 486-based PC under DOS with GAMS 2.25 and MINOS5.

3. IBM RS6000 Model 530 under UNIX with GAMS 2.25 and MINOS5.

About four megabytes RAM were needed. Run times are shown in Table 1.

D. COMPARISON WITH CURRENT SYSTEM

For the data set described above the optimal solution of the Recruit Distribution Model (RDM) was obtained from MMEA. Since no complete model formulation of RDM was available to the author, only a limited comparison of our results with those of RDM is possible. We compared the achievements of the

four objectives specified by MMEA, with the results given in Table 2.

TABLE 1. RUN TIMES OF THE GAMS PROGRAM

Computer GAMS Version Solver	AMDAHL 5990 GAMS 2.19 ZOOM 2.1	IBM RS6000 GAMS 2.25 MINOS5	486 PC GAMS 2.25 MINOS5
Solution Phase 1	20 sec	57 sec	166 sec
Solution Phase 2	13 sec	9 sec	24 sec
Generation/Report	47 sec	61 sec	144 sec
Total Run Time	80 sec	127 sec	334 sec

TABLE 2. COMPARISON OF OBJECTIVE ACHIEVEMENTS

Sums of	Two-Phase Model	RDM
Assignments	460	460
Prerequisite Lev.	548	544
Waiting Time	12,376 man days	12,385 man days
Score in AACs	49,042	49,077

We can conclude that there are no significant differences between the solutions of the two models.

In both solutions 460 recruits were assigned to training classes. (One of the 461 given recruits was not eligible for any training class.) 328 recruits (71.3%) were assigned to the same training classes in both solutions. For about 60% of the remaining 132 recruits the assignments to classes were changed pairwise (e.g., in our solution recruit i is assigned to class j and recruit l to class k, while RDM assigns recruit i to class k and recruit l to class j) without any change in the objective function value.

Out of 23 priority groups 15 were filled with the same number of recruits in both solutions, six groups had a fill difference of one, and two groups had a fill difference of two. No tendency was observed that one of the models achieved a significant better fill of groups with high priorities (see Appendix).

VI. CONCLUSIONS AND RECOMMENDATIONS

This thesis describes a two phase optimization model for the assignment of Marine recruits to occupational training. In phase one an elasticized linear program with relaxed integrality constraints is solved to calculate the optimal fill of the training classes. These optimal values are used to compute integer lower and upper bounds on the fill of the classes for a network model which is solved in phase two, yielding an integer solution.

A. CONCLUSIONS

In Chapter V it was shown that the solutions of the two phase model are very similar to those of the Recruit Distribution Model which satisfy the requirements of the Marine Corps. The results look promising for using our model as a basis for an optimization system to assign recruits to training classes. The model provides not only an effective decision aid in the assignment process, but can also give feedback on the effect of the policies (constraints) by the elastic variables. This enables the user to modify constraint parameters in order to achieve more preferred results. The short run times of the GAMS program favor multiple runs. The implementation in GAMS makes the model independent of special hardware and allows the use of future computer environments.

It allows the model to be easily modified if new constraints are added in the future.

B. RECOMMENDATIONS

Before the model can be fully implemented as an operational tool, the preprocessing of the input data should be fully automated. In the data sources which were available to the author the qualifications of the recruits were differently coded in the recruit data records and in the Prerequisite Dictionary, e.g., "graduation from high school" is coded by "5" in the recruit data record and by "HSGRAD" in the Prerequisite Dictionary. This complicates the comparison of the data in a computer program. The coding in all data sources should be standardized. The raw data should reside in a database management system and the GAMS program should extract appropriate data and execute via DBMS commands.

The output of the GAMS program should be linked to a software package which uses the model results to write assignment orders, calculate statistics, etc..

Important objects of further research are the weights of the objective functions. The idea of combining objectives through a weighting function is equivalent to assuming a linear utility function [Ref. 2]. A nonlinear utility function could be found which more accurately reflects MMEA's preferences for making trade-offs between the various constraints and objective functions. In phase one a nonlinear

program with this nonlinear utility function as objective function can be solved as done by Harrison and Rosenthal [Ref. 9], while in phase two the network can still be used to get integer solutions. Further improvement of the results is possible with this enhancement.

APPENDIX. COMPARISON OF CLASS FILL

TABLE 3. COMPARISON OF FILL BY PRIORITY GROUPS

Priority	FILLNPS	FILLRDM	Difference
3	3	3	
4	1		1
6	1	1	
8	7	7	
9	5	5	
10	30	30	
11	4	4	
12	184	185	-1
13	30	29	1
14	7	9	-2
16	10	10	
17	22	22	
18	6	6	
19	5	6	-1
20	18	18	
21	19	19	
23	18	18	
24	1		1
25	16	14	2
26	7	8	-1
27	30	30	
28	3	3	
29	32	32	

FILLNPS = fill of priority groups in optimal solution of two phase model.

FILLRDM = fill of priority groups in optimal solution of RDM.

TABLE 4.   COMPARISON OF FILL BY TRAINING CLASSES

Class No.	Priority	Quota	FILLNPS	FILLRDM	Difference
28	3	1	1	1	
20	3	2	2	2	
29	4	1	1		1
47	6	1	1	1	
40	8	3	3	3	
41	8	4	4	4	
11	9	1	1	1	
37	9	4	4	4	
38	10	2	2	2	
39	10	2	2	2	
9	10	8	8	8	
10	10	18	18	18	
6	11	4	4	4	
12	12	13	13	13	
55	12	86	80	80	
58	12	99	91	92	-1
57	13	3	3	2	1
13	13	3	3	3	
60	13	3	3	3	
59	13	11	10	10	
56	13	12	11	11	
5	14	19	7	9	-2
8	16	10	10	10	
17	17	1	1	1	
14	17	21	21	21	
15	18	1	1	1	
16	18	5	5	5	
23	19	9	1	2	-1
21	19	10	4	4	
24	20	4	4	4	
25	20	4	4	4	
27	20	4	4	4	
26	20	6	6	6	
65	21	7	6	6	
64	21	7	6	7	-1
63	21	7	7	6	1
51	23	4	3	3	
52	23	6	5	4	1
49	23	7	5	5	
50	23	7	5	6	-1
53	24	2	1		1
42	25	1	1	1	
44	25	2	2		2
43	25	2	2	2	
54	25	4			

Class No.	Priority	Quota	FILLNPS	FILLRDM	Difference
45	25	5	5	5	
46	25	5	5	5	
48	25	8	1	1	
36	26	2		1	-1
35	26	2	2	2	
22	26	5	5	5	
18	27	1	1	1	
33	27	1	1	1	
34	27	1	1	1	
19	27	2	2	2	
30	27	2	2	2	
32	27	3	3	3	
62	27	4	4	3	1
31	27	5	5	5	
61	27	14	11	12	-1
7	28	6	3	3	
3	29	2	2	2	
2	29	4	4	4	
1	29	26	26	26	
4	30	3			

FILLNPS = fill of training class in optimal solution of two
phase model.

FILLRDM = fill of training class in optimal sulution of RDM.

LIST OF REFERENCES

1. Decision Systems Associates, Inc., _RDM Recruit Distribution Model - Tour-Enhanced User's Guide / Technical Report_, Decision Systems Associates, Inc., Rockville, Maryland, February 1981.

2. Rosenthal, R.E., "Principles of Multiobjective Optimization", _Decision Sciences_, v.16, pp. 133-152, 1985.

3. Bazaraa, M.S., Jarvis, J.J., and Sherali, H.D., _Linear Programming and Network Flows_, 2nd ed., p. 429, John Wiley & Sons, Inc., 1990.

4. Liebman, J.S., and others, _Modeling and Optimization with GINO_, pp. 36-37, The Scientific Press, 1986.

5. Brown, G.G., and Graves, G.W., "Design and Implementation of a Large Scale (Mixed Integer, Nonlinear) Optimization System", _ORSA/TIMS_, Las Vegas, Nevada, November 1975.

6. Witrock, R., "Scheduling Algorithms for Flexible Flowlines", _IBM Journal of Research and Development_, v.29, pp. 401-412, July 1985.

7. Brooke, A., Kendrick, D., and Meeraus, A., _GAMS: A User's Guide_, The Scientific Press, 1988.

8. Systems Exploration, Inc., _TC-EXPRESS Version of the Recruiter Allocation Model_, Systems Exploration, Inc., Monterey, California, July 1990.

9. Harrison, T.P., and Rosenthal, R.E., "A Multiobjective Optimization Approach for Scheduling Timber Harvests on Nonindustrial Private Forest Lands", _TIMS Studies in Mangement Sciences_, v.21, pp. 269-283, 1986.

INITIAL DISTRIBUTION LIST

No. Copies

1. Defense Technical Information Center   2
   Cameron Station
   Alexandria, Virginia 22304-6145

2. Library, Code 0142   2
   Naval Postgraduate School
   Monterey, California 93943-5000

3. Prof. Richard E. Rosenthal, Code 55Rl   3
   Operations Research Department
   Naval Postgraduate School
   Monterey, California 93943-5000

4. Prof. Craig A. Tovey   1
   Industrial and Systems Engineering
   Georgia Institute of Technology
   Atlanta, Georgia 30332

5. Prof. Romesh Saigal   1
   Industrial and Operations Engineering Department
   University of Michigan
   Ann Arbor, Michigan 48109

6. Commandant of the Marine Corps (MISA)   3
   attn. Maj W.T. McCullers
   Headquarters Marine Corps
   Washington, D.C., 20380-0001

7. German Military Representative USA/Canada   1
   4000 Brandywine Street, N.W.
   Washington, D.C. 20016-1887

8. Decision Systems Associates, Inc.   1
   350 Fortune Terrace
   Rockville, Maryland 20854

9. Maj Wolfgang Maskos   1
   Raketenartilleriebataillon 22
   Harthberg Kaserne
   W 3578 Schwalmstadt
   Federal Republic of Germany